

# Thermodynamics of 4D dS/AdS Gauss–Bonnet black holes according to consistent gravity theory in the presence of a cloud of strings

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## Abstract

By looking at the Lovelock theorem one can infer that the gravity model given by [1] cannot be applicable for all types of 4D Einstein–Gauss–Bonnet (EGB) curved space-time. The reason for this is that in 4D space-time, the Gauss–Bonnet invariant is a total derivative and hence it does not contribute to gravitational dynamics. Hence, the authors of [2] presented an alternative consistent EGB gravity model instead of [1] by applying a break-of-diffeomorphism property. In this work, we use the alternative model to produce a de Sitter (dS)/Anti-de Sitter (AdS) black hole metric and then investigate its thermodynamic behavior in the presence of a cloud of Nambu–Goto strings. Mathematical derivations show that the resulting diagrams of pressure vs specific volume at a constant temperature are similar to that for a van der Waals gas/fluid in an ordinary thermodynamic system in the dS sector but not in the AdS background. From this, we infer that the black hole participates in the small-to-large black hole phase transition in the dS background, while it exhibits a Hawking–Page phase transition in the AdS background. In the latter case, an evaporating black hole eventually reaches an AdS vacuum space because of its instability.

Keywords: Lovelock gravity, string, Nambu–Goto, Gauss–Bonnet, black hole, thermodynamic, cosmological constant, phase transition

(Some figures may appear in colour only in the online journal)

## 1. Introduction

Among the various higher-order derivative gravitational models described in the literature, Lovelock gravity [3] is quite special, as it is free of ghosts [4–10]. In fact, many of the higher-order derivative metric theories which have been presented exhibit Ostrogradsky instability (see [11, 12] for a good review). In this sense, the actions that contain higher-order curvature terms introduce equations of motion with fourth- or higher-order metric derivatives in which linear perturbations reveal that the graviton should be a ghost. Fortunately, the Lovelock model is free of ghost terms, and so has field equations involving no more than second-order derivatives of the metric. The action functional of Lovelock gravity is given by combinations of various terms, as

follows. The first term is the cosmological constant  $\Lambda$ , the second term is the Ricci scalar  $R = R^\mu_\mu$ , and the third and fourth terms are the second-order Gauss–Bonnet (GB) [13] and third-order Lovelock terms (see equation (22) in [14]), respectively. Without the latter term, the Lovelock gravity reduces to the simplest form called the Einstein–Gauss–Bonnet (EGB) theory, in which the Einstein–Hilbert action is supplemented with a quadratic curvature GB term as the source of the self-interaction of gravity. The importance of this form of the gravity model is more apparent when we observe that it is generated from the effective Lagrangian of low-energy string theory [15–19]. In fact, for more than four dimensions of curved space-time, the GB coupling parameter, which is calculated by the dimensional regularization method, has some