# سوال های پیشنهادی برای درس مکانیک آماری کارشناسی ارشد از فصل ۲

## (Reference: Statistical Mechanics, Third Edition, R. K. Pathria)

**2.3.** Starting with the line of zero energy and working in the (two-dimensional) phase space of a classical rotator, draw lines of constant energy that divide phase space into cells of "volume" $h$. Calculate the energies of these states and compare them with the energy eigenvalues of the corresponding quantum-mechanical rotator.

**2.4.** By evaluating the "volume" of the relevant region of its phase space, show that the number of microstates available to a rigid rotator with angular momentum $\le M$ is $(M/\hbar)^2$. Hence determine the number of microstates that may be associated with the quantized angular momentum $M_j = \sqrt{\{j(j+1)\}}\hbar$, where $j = 0, 1, 2, \ldots$ or $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$. Interpret the result physically.

[*Hint*: It simplifies to consider motion in the variables $\theta$ and $\varphi$, with $M^2 = p_\theta^2 + (p_\phi/\sin\theta)^2$.]

**2.6.** The generalized coordinates of a simple pendulum are the angular displacement $\theta$ and the angular momentum $ml^2\dot{\theta}$. Study, both mathematically and graphically, the nature of the corresponding trajectories in the phase space of the system, and show that the area $A$ enclosed by a trajectory is equal to the product of the total energy $E$ and the time period $\tau$ of the pendulum.

**2.7.** Derive (i) an *asymptotic* expression for the number of ways in which a given energy $E$ can be distributed among a set of $N$ one-dimensional harmonic oscillators, the energy eigenvalues of the oscillators being $\left(n + \frac{1}{2}\right)\hbar\omega; n = 0, 1, 2, \ldots$, and (ii) the corresponding expression for the "volume" of the relevant region of the phase space of this system. Establish the correspondence between the two results, showing that the conversion factor $\omega_0$ is precisely $h^N$.

**2.9.** **(a)** Solve the integral

$$\int_{0 \le \sum_{i=1}^{3N} |x_i| \le R} \cdots \int (dx_1 \ldots dx_{3N})$$

and use it to determine the "volume" of the relevant region of the phase space of an extreme relativistic gas ($\varepsilon = pc$) of $3N$ particles moving in one dimension. Determine, as well, the number of ways of distributing a given energy $E$ among this system of particles and show that, asymptotically, $\omega_0 = h^{3N}$.

**(b)** Compare the thermodynamics of this system with that of the system considered in Problem 2.8.